On the nature of Minimal Search

Sandiway Fong
University of Arizona
`sandiway.arizona.edu`

1. Why study Search?

- Search applies everywhere.
- Structural dependencies all involve search, e.g. (1) an adverb must find a verb to modify, (2) an anaphor an antecedent, (3) EXT economy structurally identical inscriptions, and (4) INFL a θ-relevant item.
- Under the SMT, we want the simplest possible formulation of **Minimal Search (MS)**. Chomsky argues there are no exceptions, e.g. inputs to Merge also obey **MS**.
- The complexity of Search, i.e. in what it can find, directly affects the complexity of human language.

MS is a fundamental operation that underpins nearly everything:

- *Right now, I don't see any reason why any operation should be exempt from* **MS**. (Chomsky, p.c.)

Search of I-Language structures can be carried out in many possible ways, but the **Third Factor** limits the complexity of possible search operations.
Optimal case: there is only one search, the simplest possible search (which is also the most efficient one).

2. Background

SMT (*Strong Minimalist Thesis*): a reduction to only fundamental operations needed to **construct** and **parse** thought structures, *deep simplicity* motivated by evolutionary recency of human language and the slow (chemical) brain (**Third Factor**).

construct: **Merge** (External & Internal) and **FormSet**. (Pair Merge needed anymore?)
parse: (structural) relations establish by search through I-Language structures, suppose only **Minimal Search (MS)** is available for this.

Merge:
A mathematical core (*minimum necessary to build structure*), and
a language organ (**LSC**) variant *(includes θ-theory etc.)*.

Three Factors: First, *genetic endowment*. Second, *experience* (Chomsky 2005).

Third Factor: computational efficiency imposed by biological limits, e.g. brain's 20W power consumption.

- *To what extent is the human faculty of language FL an **optimal solution** to minimal design specifications, conditions that must be satisfied for language to be usable at all?* (Chomsky 2001)

Optimal case: what's most simple is also the most efficient.

[**term-of**: recursively defined subset of set. Sometimes reflexive as well.]

3. MS and Merge

1. MS applied to External Merge (EM)

Search: domain: Workspace (WS), target: *WS item* (*twice*)
Formally: find X,Y ∈ WS, form {X, Y}

MS: can't search inside X (or Y), must stop at *WS item*.
       *EM': form {x, Y} st. x term-of X, X, Y ∈ WS, or
       *EM": form {x, y} st. x term-of X, y term-of Y, X,Y ∈ WS

LSC:
- *theta structures {X,Y}, where one member receives and the other assigns a theta role, an LSC* (Chomsky 2024)

2. MS applied to Internal Merge (IM)

Search domain: step 1) WS, step 2) WS item
Search target: step 1) *WS item*, step 2) term-of *WS item*
Formally: 1) find X ∈ WS, 2) find Y term-of X, form {X, Y}

MS: 1) can't search inside X
MS: 2) Y must be the highest occurrence of Y in X.
       {**Y**, {..{Y..}}}   (*cannot tell structurally identical inscriptions apart*)
       {**Y**$_\theta$, {Z$_\theta$,..}}   (θ-relevant Y and Z)
       Equidistance, e.g. specifiers (Chomsky 1995)

LSC: in 2), target θ-relevant item

4. Simplest Search = MS

***Simplest possible* notion of Search**:
look for something, find first matching item, terminate search. Minimal computation.

EM brings structural relation **sister-of**:
In {X, Y}, X and Y, inputs to EM, are sisters.

IM brings structural relation ***term-of***:
In {..{X,..}} = Y, X and Y inputs to IM, X is a (proper) term of Y

EM + IM brings composite structural relation: ***term of sister*** (**c-command**)

Mathematically, *term of sister*:
1. a. {{X, Y}, {U, {V, W}}} has c-command relations:
   b. (X, Y) and (Y, X),
      (U, V), (U,W),
      (V, W) and (W, V)   (6 total)

Merge of **Z** to (1a) results in (2a):
2. a. {**Z**, {{X, Y}, {U, {V, W}}}} adds c-command relations:
   b. (Z, X), (Z, Y), (Z, U), (Z, V), (Z, W)   (5 new relations)

MS is efficient: do not (pre-)compute all possible c-command relations.

MS is limited by Phase boundaries? (Strong *Third Factor)*. No, except perhaps in EXT.

More complex notions that are ruled out because they're too (computationally) expensive.
(Design Principle – *costless* (or cost paid upfront) vs. Filter – a *cost each time used*.)

Sorting, i.e. search with comparison:
e.g. find two candidates, compare, select "best" one. (*Optimality Theory*)

Multi/partial-probing:
e.g. T_ϕ probes EXPL (incomplete-φ) and R-expr (complete-φ) (*DbyP*)

3. a. *there is likely to arrive a man*
   b. {T_ϕ, {v, {likely, {EXPL, {v, {arrive$_\theta$, man$_a$}}}}}}
   c. relations: (T_ϕ, EXPL) (T_ϕ, man$_a$)

Non-determinism:
e.g. {..{X, Y}..}, (distinct) X & Y both possible targets. Maybe there are no such configurations. Suppose I-Language is organized so MS never encounters non-determinism.

4. a. a rose is a rose   (Moro)
   b. {INFL$_\phi$, {be, {rose$_a$, rose$_a$}}}   (*apparent non-determinism*)

   • *we have to [distinguish substantive-predicative], perhaps resorting to the categorization in terms of ±substantive, ±predicative that we've discussed.* (Chomsky, p.c.)

Asymmetry (Moro 2017):

5. a. John considers a picture of the wall (to be) the cause of the riot
    b. John considers the cause of the riot *(to be) a picture of the wall

Basic θ-configurations

- *If the propositional core of language keeps to binary theta structures, there will be no n-ary theta structures for n>2. In particular, complex VPs such as double-object constructions will have internal structure, as commonly assumed.* (Chomsky 2024)

We assume θ-configurations are basic/universal.
Each functional head v, R, Appl may introduce a single θ-relevant item.

6. a. $\{v, \{R_\theta, IA\}\}$ (unaccusative)
    b. $\{EA, \{v_\theta, R\}\}$ (unergative)
    c. $\{EA, \{v_\theta, \{R_\theta, IA\}\}\}$ (transitive)
    d. $\{EA, \{v_\theta, \{IA_1, \{Appl_\theta, \{R_\theta, IA_2\}\}\}\}\}$ (ditransitive: high applicative)
    e. $\{EA, \{v_\theta, \{IA_1, \{R_\theta, Appl_\theta,, IA_2\}\}\}\}$ (ditransitive: low applicative)

(Possibility to not introduce EA in (6c–e) if v instead of v_θ.)
Binary Merge ensures arguments (EA/IA) are always asymmetric wrt. **MS**.

Merge is θ-aware (and *selection-aware*). Build (6a–e) efficiently, e.g. no

7. *$\{EA, \{R_\theta, \{v_\theta, IA\}\}\}$

Assume INT can directly decode/read θ-configurations: no need for (earlier) θ-assignment operation.

Assume MS can also efficiently decode these structures (*see also Appendix*).

Where are the results of MS stored?

Relations computed by MS on I-Language structures must be stored somewhere.

Relations must be accessible for processes downstream from Merge computation:
    (i)   semantic interpretation (*dependencies*), and
    (ii)   EXT (*pronounce which occurrences and where*?).

Not stored in Merge structure: no room for it (or labels).
Not stored in the Workspace: assume all WS objects are subject to Merge.
Assume WS computation is complete when a single WS object is left.
For convergence, *all* WS objects must be used, can't have MS relations around.

Example: XP will be assembled in a sub-WS for {XP, YP}, XP = EA and YP = vP.
    8. [The man I knew] found the artifact

Sub-WS vanishes once complete.

5. MS and INFL

3. MS for INFL$_\phi$
Transitioning from the proposition to the clausal domain involves Merge of INFL (and C).
INFL$_\phi$ triggers search for a θ-relevant item. Only search possible is **MS**.

9. a. {INFL$_\phi$, {EA, {v$_\theta$, {R$_\theta$, IA}}}}
    b. ɸ-relation (INFL$_\phi$, EA)

In (7a), EA is found, IA is never seen. See (6a–b,d–e) for other cases.

At INFL$_\phi$ EXT, verb agreement obtains and a range of (language-particular) spell-out
options are possible for the found item, e.g. in English:

10. a. He$_\phi$ *repair/repairs$_\phi$ the cars   (left edge INFL; EA Agree)
    b. Many soldiers$_\phi$ *has/have$_\phi$ arrived   (left edge INFL; IA Agree)
    c. There *has/have$_\phi$ arrived many soldiers$_\phi$ (no edge spellout; IA Agree)
    d. We should do this again, said$_\phi$ John$_\phi$ (right edge INFL)
    e. In the distance stands$_\phi$ a mountain range$_\phi$ (no edge spellout; EA Agree)
    f. In this cave was$_\phi$ found an ancient artifact$_\phi$ (no edge spellout; IA Agree)
    g. There entered$_\phi$ the room a strange man$_\phi$ (right edge of INFL: TEC)
    h. There hit$_\phi$ the stands a new journal$_\phi$ *ditto*. Kayne in (Chomsky, 2001)

(8c,g–h) EXPL insertion. (Not available for all languages.)
(8d) quotative inversion.
(8f–g) locative inversion.
(8d,g–h) right edge of INFL$_\phi$P spellout. (Spanish has subject-verb inversion.)

*   *Since SPEC-INFL is not at the phase level, it is not accessed at later phases.
    Therefore, it need not to be boxed. And it shouldn't be, since unlike boxed
    elements, SPEC-INFL is accessible to other operations (Agree, Labelling,
    Anaphora)*   (Chomsky 2024)

(Chomsky 2021) notes that there are semantic, i.e. non-stylistic, effects, if argument
spellout is at the left edge of INFL.

11. a. *There* is *a* flaw in my argument/bottle
    b. *A flaw is in my argument
    c. A flaw is in my bottle
    d. {{INFL$_\phi$, {v, {R, IA$_\phi$}}}, in my argument/bottle}

there-existentials generally signal new information, which interacts with *definiteness*
(Milsark 1974).

12. a. *there* arose *a*/*the storm
    b. *a*/*the* storm arose (last night)

This kind of information will be coded differently depending on particular language, which suggests this also belong to EXT (*directly signaling C-I interface*).

Chinese has no expletive *there*; instead, signal is encoded via word order variation (Wu 2020, *citing earlier work*).

13. a. 老师　　来了
    teacher　come.PERF　　　　　(old)
    '*the* teacher came'
    b. 来了　　　一个　　老师
    come.PERF　one.CLF teacher　　(new) cf. *那个老师 / that.CLF teacher
    '*there* came *a* teacher'

In Spanish, the presence of impersonal verb *haber* (*have*) signals the *there*-existential:

14. a. *un* policía　　está aquí / *hay*　*un* policía aquí　　　(Tubens, p.c.)
    a　policeman is　here　there.is a　policeman here
    'a policeman is here' / 'there is a policeman here'
    b. (*había)　　surgió *una* tormenta
    there.was arose　a　　storm
    'a storm arose'

6. MS and Modification

The adverb *carefully* seeks a verb in (15a) (Chomsky, 2021). Sentence is structurally ambiguous.

15. a. the mechanic who fixed the car *carefully* packed his tools
    b. relation (carefully, fix), right edge vP spellout, or
    c. relation (carefully, pack), left edge vP spellout

MS applies to compute relations (9b–c) from (10a–b), respectively.

16. a. {mechanic, {v_θ, {*carefully*, {fix_θ, car}}}}, or
    b. {mechanic who ..., {v_θ, {*carefully*, {$pack_\theta$, his tools}}}}

(16a–b) can also be spelled out as unambiguous (17a–b), respectively.

17. a. the mechanic who *carefully* fixed the car packed his tools
    b. the mechanic who fixed the car packed his tools *carefully*

Note 1: adverb *carefully* appears in a sub-WS in (16a), but in the main/matrix WS in (16b).

Note 2: structures (16a–b) assume θ-theory has primacy. $\{R_\theta, IA\}$ must be built first.

Adverbs (seeking verbs) are not part of θ-Merge, but MS still generally applies to WS Merge (Note: no Pair Merge).

7. MS and EXT

Economy of spellout applies at EXT
This also involves MS within a Phase, e.g. for c-commanded identical inscriptions, only the highest occurrence is pronounced.

In (18a) below, Phase = **{ ... }**, see fn.30 in (Chomsky 2024).
(TNS, as part of v, not shown.)

18. a. $\{INFL_\phi, \{$John, **{**$v_\theta, \{see_\theta,$ John$\}$**}}** $\Rightarrow$ *John saw John*, not *John saw*
    b. $\{INFL_\phi, \{v, \{arrive,$ John$\}\}\}$ $\Rightarrow$ *John arrived*
    c. $\{INFL_\phi, \{$most people, $\{v_\theta, \{try, \{v, \{honest,$ most people$\}\}\}\}\}\} \Rightarrow$ *Most people try to be honest*

For (18c), copy relation (most people, ~~most people~~) computed by MS and used at EXT.

8. MS and FormSet

- *The only other permissible relation is unbounded set, with the SBO FormSet (**FS**)* (Chomsky 2024)

What's different about the fundamental *Structure Building Operation* (SBO) **FS**, cf. Merge? **Answer**: they have different conditions for formation and use, no overlap.

For FS, the minimal assumptions (Fong & Oishi 2025):
   (i)   Members must be a coherent set of WS objects
   (ii)   Members must obey some parallelism requirement for INT
   (iii)   **MS** into FormSet, all members must be treated alike

MS applies to FS inputs just like in the case of EM, i.e. inputs must be objects, not sub-objects of the WS.

(19b) and (19c) below are *coherent* WS objects, both are vP θ-configurations.
FS applied to (19b–c) produces (19d), also a WS object. (Notation: FS object **{...}**.)
Merge of INFL triggers MS. Following (iii) above, MS finds the identical inscription *John* in both members.

19. a.   John arrived *and* met Bill
    b. $\{v, \{arrive,$ John$\}\}$
    c. $\{$John, $\{v^*, \{meet,$ Bill$\}\}\}$
    d.   **{**$\{v, \{arrive,$ John$\}\}, \{$John, $\{v^*, \{meet,$ Bill$\}\}\}$**}**
    e. $\{INFL_\phi,$ **{**$\{v, \{arrive,$ John$\}\}, \{$John, $\{v^*, \{meet,$ Bill$\}\}\}$**}}**

Economy of spellout applies for (19e), lower occurrences of *John* are not pronounced.

When MS cannot operate identically across members, as in the case of (20b), the derivation crashes.

20. a. John arrived and Sally met Bill
    b. *{INFL$_\phi$, {{v, {arrive, John}}, {Sally, {v*, {meet, Bill}}}}}
    c. {{C, {INFL$_\phi$, {v, {arrive, John}}}}, {C, {INFL$_\phi$, {Sally, {v*, {meet, Bill}}}}}}

Instead, (20a) is derived via (20c).

Note in the case of *serial verbs constructions* (SVCs), e.g. in Japanese or Chinese, independent clausal structure, as in(20a), is impossible. These are monoclausal constructions with verbs that *must* share (some) argument structure.

In (21a) below, the two SVC verbs are transitive, sharing both arguments.

21. a. Ken-ga   Miki-o   home-tatae-ta
         K.-NOM M.-ACC admire-praise-PST
        'Ken greatly admired Miki'
    b. {EA, {v$_\theta$, {{admire$_\theta$, praise$_\theta$}, IA}}}

In (21b), EM of IA invokes MS for each FS member, a predicate, to find the identical inscription *Miki* (= IA) in the WS.
Note that EM of *Ken* (=EA) does not involve FS.

It is also possible that SVC verbs have independent IAs. For example, in (22a), the verbs *nomu* (*drink*) and *aruku* (*walk*) have independent IAs, *sake* (*alcohol*) and *yoru no mati*, respectively, but share EA *Ken*. In (22b), MS finds each IA independently prior to FS.

22. a. Ken-ga yoru-no   mati-o   sake-o   nomi-arui-ta
         K.-NOM night-GEN town-ACC alcohol-ACC drink-walk-PST
        'Ken went bar-hopping at night'
    b. {EA, {v$_\theta$, {{drink$_\theta$, *sake*}, {walk$_\theta$, *yoru-no mati*}}}}

[Note: Japanese examples (21a) and (22a) above taken from unpublished joint work with J. Ginsburg, H. Terada & M. Matsumoto.]

Assuming Chinese *qu* (go) and *gen* (follow) are both SVC verbs that share an EA in example (23a), the corresponding structure is given in (23b).

23. a.   Zhangsan zhunbei gen   ni   qu   (Huang, 1989)
          张三   准备   跟   你 去
                prepare follow you go
         'Z. plans to go with you'
    b. {EA, {v$_\theta$, {go, {follow$_\theta$, IA}}}}

9. A second form of MS

My claim is that there is another MS predicted by SMT.

Recall that IM brings term-of and EM brings sister-of.

- *Relations are binary. The simplest ones are sisterhood and term-of, the domains of EM and IM, respectively*. (Chomsky 2024)

A (multi-valued) function is an input-to-output mapping of a relation.
(Top-down) MS is functional composition Y = term-of(sister-of(X)).

Sister-of is symmetric, but term-of isn't.
Define, if X term-of Y, Y term^{-1}-of X.

Bottom-up MS is just Y = sister-of(term^{-1}-of(X)).
Of course, term^{-1}-of is just *dominates*.

Exactly what we need for anaphors, which generally seek a closest antecedent.
Generally, we have a need for both top-down and bottom-up MS.

So-called *each*-insertion (Chomsky 1973):

24. a. The men *each* hated the *other*
    b. *Each* of the men hated the *other*
    c. The men hated *each other*
    d. *The men hated the *other*

MS: *each* must quantify over *men*, *other* seeks *each*.
Relation (*each*, *men*) is found by top-down MS and (each, other) by bottom-up MS (at INT) for interpretation purposes.

25. a. {INFL$_\phi$, {*each*, {*men*, {v$_\theta$, {hate$_\theta$, *other*}}}}}}
    b. {INFL$_\phi$, {{*each*, {of, *men*}}, {v$_\theta$, {hate$_\theta$, *other*}}}}

[We employ the color *green* to indicate where bottom-up search will be triggered, and *blue* for top-down.]

26. a. *Neither* of the men hated the *other*
    b. *Neither* man hated the *other*
    c. ??*Neither* man *each* hated the *other*

Following (Chomsky 2021), raising to object (of *expect*) of the embedded subject:

27. a. The *candidates each* expected the *other* to win
    b. The *candidates* expected *each other* to win

28. a. {INFL$_\phi$, {*each*, {*candidates*, {v$_\theta$, {*other*, {expect$_\theta$, …}}}}}}
    b. {INFL$_\phi$, {*candidates*, {v$_\theta$, {*each other*, {expect$_\theta$, …}}}}}

Search is always targeted, search initiated by *other* skips the NP *candidates* in (22a).

Does bottom-up MS go beyond the local clause?
- *anaphora is not generally restricted by PIC* (Chomsky 2024: fn.29)

(Chomsky 2024) θ- and clausal configurations:

29. a. [which pictures of each other] did Mary think they hope Bill likes?   (*adapted*)
    b. θ: {Mary, {v$_\theta$, {think, {they, {v$_\theta$, {hope, {Bill, {v$_\theta$, {like$_\theta$, wpeo}}}}}}}}}} (Search in θ)
    c. C: {C$_Q$, {…, {v$_\theta$, {think, {C, {INFL$_\phi$, {…, {v$_\theta$, {hope, {C, {INFL$_\phi$, {…, {v$_\theta$, {like$_\theta$, wpeo}}}}}}}}}}}}}} (Search in C)
    d. {C$_Q$, {Mary, {v$_\theta$, {think, {C, {INFL$_\phi$, {they, {v$_\theta$, {hope, {C, {INFL$_\phi$, {Bill, {v$_\theta$, {like$_\theta$, wpeo}}}}}}}}}}}}}} (*combined*)

EXT: C$_Q$; anaphor: upward search to *they* (target: plural noun)
Other clausal heads, e.g. Topic, Rel (*relativization*): *search can find relevant item* …

Lower object is not subject to raising to matrix object.

30. a. The candidates expected to defeat *each other*
    b. The candidates *each* expected to defeat the *other*

Other specifically targets *each*, and *each other* targets the nearest θ-relevant binder.

31. a. The men *each* expected the soldier to shoot the *other*
    b.   *The men expected the soldier to shoot *each other*

32. a. The men *each* saw John's pictures of the *other*
    b. *The men saw John's pictures of *each other*

33. The men *each* believed and expected the sniper to try to target the *other*

Appendix: MS and Decoding of Structure

[*Much to say here, no time left*.]

Briefly, MS terminates when it sees the first item detected.
Target a θ-relevant item:

… pictures of themselves   (*of*-insertion)
{pictures$_{the}$, themselves}   detect *pictures*

Must be able to distinguish predicate/argument generally. In:

{XP, YP}   (predicative vs. substantive)

It must know which branch, XP or YP, to search.

Psycholinguistic experiments indicate asymmetries arise for degraded acceptability of *wh*-fronting in subject vs. object. E.g. "Superiority" or " *wh*-Object Penalty" in Spanish. Some report a "*wh*-Subject Penalty" in Spanish.

REFERENCES

Chomsky, N. A. (1973). Conditions on Transformations. In: Anderson and Kiparsky (eds.), *A Festschrift for Morris Halle*, New York: Holt, Rinehart & Winston, pp. 232–286.

Chomsky, N. A. (2005). Three Factors in Language Design. *Linguistic Inquiry*, 36:1, pp. 1-22.

Chomsky, N.A. (2021). Minimalism: Where are we now, and where can we hope to go. *Gengo Kenkyu* 160, pp. 1-41.

Chomsky, N.A. (2024). The Miracle Creed and SMT. In: M. Greco and D. Mocci (eds.), *A Cartesian Dream: A Geometrical Account of Syntax. In Honor of Andrea Moro*. Rivista di Grammatica Generativa/Research in Generative Grammar Monographs, pp. 17-40.

Fong, S. & M. Oishi (2025). On the Nature of FormSet. In *Language Variation*. https://doi.org/10.1075/lv.24032.fon

Huang, C.-T. J. (1989). Pro-drop in Chinese: A Generalized Control Theory. In: 0. Jaeggli and K. Safir (eds.), *The Null Subject Parameter,* pp. 185-214.

Milsark, G. L. (1974). *Existential Sentences in English*. Ph.D thesis. MIT.

Moro, A. (2017). In M. Everaert and H.C. van Riemsdijk (eds.), The Wiley Blackwell Companion to Syntax.

Wu, L. (2020). L1 Transfer in L2 Acquisition of the There-Insertion Construction by Mandarin EFL Learners. *International Journal of Foreign Language Teaching & Research*. Vol. 8, Issue 31.